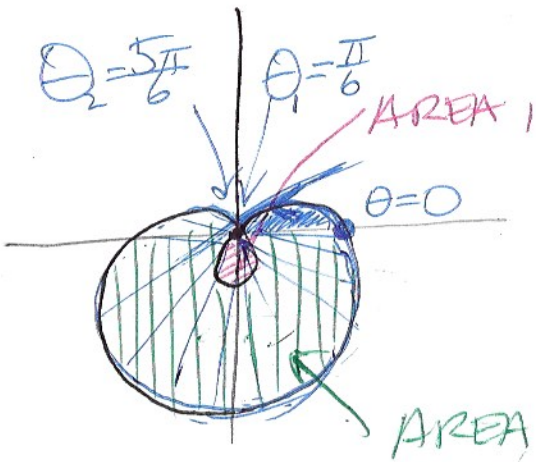
$r = 1 - 2\sin\theta$

AREA INSIDE LIMACON =



AREA INSIDE INNER LOOP

$+$

AREA INSIDE OUTER LOOP AND OUTSIDE INNER LOOP

$$\int_0^{\frac{\pi}{6}} \frac{1}{2}(1-2\sin\theta)^2 d\theta + \int_{\frac{5\pi}{6}}^{2\pi} \frac{1}{2}(1-2\sin\theta)^2 d\theta$$

$\theta = 0 \rightarrow r = 1 - 2\sin 0 = 1 \quad (1,0)$

$r = 1 - 2\sin\theta_1 = 0$

$\sin\theta_1 = \frac{1}{2} \qquad \theta_1 \in [0, 2\pi]$

$\theta_{REF} = \frac{\pi}{6}$

~~$\frac{\pi}{6}$ $\frac{\pi}{6}$~~

$\theta_1 = \frac{\pi}{6}, \frac{5\pi}{6}$

$\int \sin^2\theta d\theta = \frac{1}{2}(\theta - \frac{1}{2}\sin 2\theta)$

$\int \cos^2\theta d\theta = \frac{1}{2}(\theta + \frac{1}{2}\sin 2\theta)$

$\int \sin\theta\cos\theta d\theta$ OR $u = \sin\theta$

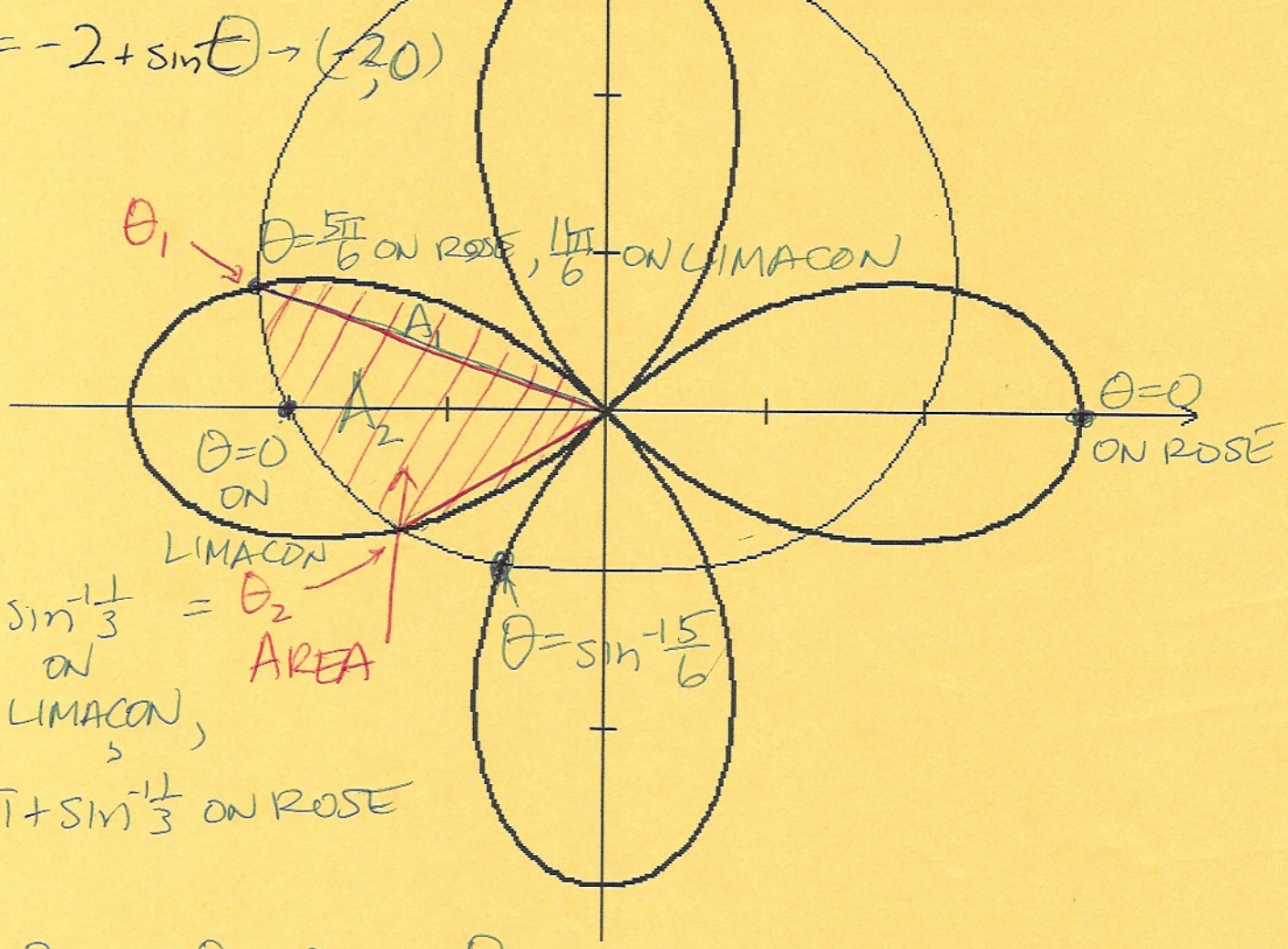
$= \int \frac{1}{2}\sin 2\theta d\theta$ $du = \cos\theta d\theta$

$= -\frac{1}{4}\cos 2\theta$ $\int u du$

$= \frac{1}{2}u^2$

$= \frac{1}{2}\sin^2\theta$

$r = -2 + \sin\theta \to (-2, 0)$



$\sin^{-1}\frac{1}{3} = \theta_2$ ON LIMACON, AREA

$\pi + \sin^{-1}\frac{1}{3}$ ON ROSE

$$3\cos 2\theta = -2 + \sin\theta$$

$$3(1 - 2\sin^2\theta) = -2 + \sin\theta$$

$$3 - 6\sin^2\theta = -2 + \sin\theta$$

$$0 = 6\sin^2\theta + \sin\theta - 5$$

$$0 = (\sin\theta + 1)(6\sin\theta - 5)$$

$\sin\theta = -1$ or $\sin\theta = \frac{5}{6}$

$\theta = \sin^{-1}\frac{5}{6}$ $\quad \sin^{-1} x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$r = 3\cos 2\theta \xrightarrow{(-r,\pi+\theta)} -r = 3\cos 2(\pi+\theta) = 3\cos(2\pi+2\theta) = 3\cos 2\theta$

$r = -2+\sin\theta$ $\qquad r = -3\cos 2\theta$

$3\cos 2\theta = -2+\sin\theta$

$3(1-2\sin^2\theta) = -2+\sin\theta$

$3-6\sin^2\theta = -2+\sin\theta$

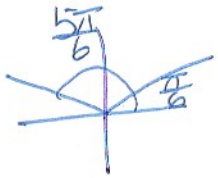
$6\sin^2\theta-\sin\theta-1=0$

$(3\sin\theta+1)(2\sin\theta-1)=0$

$\sin\theta = -\frac{1}{3}$ or $\sin\theta = \frac{1}{2}$

$\theta_{REF} = \sin^{-1}\frac{1}{3}$ $\qquad \theta_{REF} = \frac{\pi}{6}$

$\theta = \pi+\sin^{-1}\frac{1}{3},\ 2\pi-\sin^{-1}\frac{1}{3}$ $\qquad \theta = \frac{\pi}{6}, \frac{5\pi}{6}$



$3\cos 2\theta = 0$

$\cos 2\theta = 0 \qquad \theta \in [0,2\pi]$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \qquad 2\theta \in [0,4\pi]$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$$A_1 = \int_{\frac{3\pi}{4}}^{\frac{5\pi}{6}} \frac{1}{2}(3\cos 2\theta)^2 d\theta$$

$$A_2 = \int_{\frac{11\pi}{6}}^{\sin^{-1}\frac{1}{3}} \frac{1}{2}(-2+\sin\theta)^2 d\theta$$

$$\int_0^{\sin^{-1}\frac{1}{3}} \frac{1}{2}(-2+\sin\theta)^2 d\theta + \int_{\frac{11\pi}{6}}^{2\pi} \frac{1}{2}(-2+\sin\theta)^2 d\theta$$

$A_3 =$